Applying Principles of Instructional Development To Problem Solving: A Case Study

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When a teacher identifies a task which cannot be handled automatically in the daily routine, it is appropriate to approach the problem from a higher conceptual level. Such was the case with the author's attempts over the past ten years to improve the problem-solving skills of several groups of students. The thrust of this study was twofold: to develop skills in instructional design and to use these skills to develop an effective approach to teaching problem solving. The first entailed adapting tools of instructional design to the character of the author. The second entailed assembling a framework for understanding the anatomy and physiology of problem solving, practising the diagnosis of problem solving difficulties, and developing effective prognoses.

Consider the following example from daily life. A husband was required to unpack and assemble a new vacuum cleaner according to the manufacturer's directions. The simple, structure-blind solution, following the instructions, was rejected by the husband, who dislikes following written instructions (emotional block). He felt that if the machine were well designed, he should be able to assemble it. Two sections of the handle had to be bolted in place. The first was to be attached to two pre-drilled brackets mounted on the machine. There was some difficulty in determining which bracket should take the bolt and which, the threaded collet into which the bolt was to be fastened. There was also some puzzlement concerning the orientation of the handle section. Eventually it was fastened securely. The next task was to bolt the second section in place. Its orientation was dictated by the bend at the top which had to face back to serve as a handle (correct solution dictated by proper function). At this point the wife, who had begun to supervise, pointed out that the cord cleats of the two handle sections were not aligned. Both sections were removed and the lower one rotated. Finally the two sections were mounted correctly and the dust bag was hooked in place. The husband noticed to his dismay that a piece, whose function was to cover the mounting brackets for appearance sake, was lying on the floor (failure to use all the given). Finally the handle was disconnected at the base, the additional piece inserted, and the handle reattached. Why did the husband not seek through simulation (laying out all the parts in order and imagining the assembly procedure) to plan his solution? Why did he focus on one subproblem (bolting the first section in place) without considering its relations to the whole?

A key hypothesis in this study is that conceptual tools, applicable to a wide range of problems, can be developed through instruction. It is also assumed that the automatic employment of these tools will be enhanced through directed practise.

Attention was restricted to formal problems in mathematics and logic. Such problems have well-defined given elements, conditions, and solutions. Some attention was also given to problem finding as a method of exploring uncharted intellectual territory.

The study was conducted in an actual teaching situation. The author was employed by the University of Saskatchewan as a Sessional Lecturer for the Department of Mathematics during Summer Session 1980. The course taught was Math 20[illegible] An Introduction to Modern Mathematics. The course was attended by teachers and students in the College of Education.

Rationale

There can be no question that everyone has need for the capability of flexible, independent thinking. If problem solving skills are highly developed in an individual he will be more competent to deal with new and challenging situations and will have a stronger will to see himself through difficulties. Such a person will be intrinsically motivated.

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The joy of discovery and the opportunity for the teacher to foster it are stated by the mathematician and teacher whose problem-solving model is at the root of this study (Polya, 1957).

> *A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime.*
>
> *Thus a teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking.* (p. [illegible])

It is assumed in this study that problem solving skills in mathematics can be acquired, can be taught, and are better learned if explicit instruction is designed. The author has noticed a definite improvement in his own ability to solve problems as he has studied the process and attempted to teach in recent years. There is increasing evidence that a scientific approach to the teaching of problem solving can lead to significant improvements (Reif, 1981).

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Problem solving skills are of double significance to teachers. Increasing demands on teachers to produce learning outcomes related to properly constructed objectives necessitate a systematic problem-solving approach. A coherent program must be built within the framework of a society of rapidly increasingly complexity and change. The teacher must also prepare his students for this world and hence must in some way teach problem solving.

Problem Solving

The basis for the approach to problem solving adopted in this study is the following list of questions elaborated by Polya in this dictionary of heuristic (Polya, 1957).

Understanding the Problem

First. You have to understand the problem.

What is the unknown? What are the data? What is the condition?

Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

Devising a Plan

Second. Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan of the solution.

Have you seen it before? Or have you seen the same problem in a slightly different form?

Do you know a related problem? Do you know a theorem that could be useful?

Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.

Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?

Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

Carrying out the Plan

Third. Carry out your plan.

Carrying out your plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?

Looking Back

Fourth. Examine the solution obtained.

Can you check the result? Can you check the argument?

Can you derive the result differently? Can you see it at a glance? Can you use the result, or the method, for some other problem? (pp. 16-17)

The work of all other authors consulted can be related to some aspect of Polya's model. Adams is concerned primarily with conceptualization. Conceptual blocks are "mental walls which block the problem-solver from correctly perceiving a problem or conceiving its solution" (Adams, 1974, p. 11). He analyses several common types of blocks — perceptual, cultural and environmental, emotional, and intellectual and expressive. These blocks may be overcome through analysis and solution of selected problems. He emphasizes "understanding the problem" and "devising a plan".

According to Reif, the teacher can improve his teaching of problem solving if he studies the nature of the cognitive skills of individual students. Effective problem solving requires three prerequisites (Reif, 1981):

1. an effective strategy for breaking a problem into simpler, readily solvable problems;

2. a suitably selected repertoire of readily solvable problems to serve as building blocks;

3. a carefully organized knowledge base.

To attain the first of these, one should describe the problem in familiar terms. The breakdown of the problem is done most effectively by successive refinement, just as an artist would begin with an outline, add structural details, and end with those refinements which complete and perfect the work. The knowledge base and repertoire of simpler problems should be organized hierarchically for better recall and adaptation. Logical components such as definitions, principles, symbols, and equations must be acquired through wide experience to give them functional value. (This was of great importance in this study). Finally, Reif emphasizes the importance of revising and evaluating any solution.

Often, in spite of systematic effort, the solution falls into place suddenly and unexpectedly. It is as if the subconscious were working on the problem all along and suddenly presented its results. Such reactions are referred to by psychologists as "aha! reactions". Martin Gardner has written a delightful book in the belief that one can develop unconventional nonlinear thinking through practice. He is convinced of the significance of such thought (Gardner, 1978):

> *There certainly is a close connection between aha! insights and creativity in science, in the arts, business, politics, or any other human endeavor. The great revolutions in science are almost always the result of unexpected intuitive leaps. . . In many cases the solution is not found by exhaustive trial and error. . . . In many cases the solution is a Eureka insight. (p. 7)*

Throughout the writing of the authors discussed one can find implications of Gestalt psychology (Wertheimer, 1945). Or-

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ganizing knowledge hierarchically, avoiding a blind trial-and-error approach, separating periphera from essential details, seeking relations between the parts, considering the function of each step, and thinking in steps which lead to structural improvement are all keys to productive thinking. All authors agree that these elements can be developed through practise and good teaching.

Instructional Design

In order to improve his teaching of problem solving, the author elected to use current ideas on instructional design, integrated with his own model for producing a teaching/learning system. A systematic approach was needed to identify, solve, and evaluate instructional problems.

A recently developed model, see Figure 1, offers great flexibility while keeping the whole system in focus (Kemp, 1977).

As each component is developed it may require revision of other components. For this reason, the diagram shows the elements connected in a circle and interconnected through the central feedback circle. The order of development is not determined. No element need be completely developed before proceeding to another.

The influence of systems engineering has begun to find its way into public educational circles but its full application is impractical. For the classroom teacher to specify all the components of the teaching/learning situation and to chart the interconnections would be an unwieldy task. It is more practical to extract key elements from systems analysis to produce a simple model (Gerlach, Ely, and Melnick, 1980). Such a model delineates major activities and their interconnections. It should allow for flexibility in sequencing and should provide for revision of components as the system is developed. The model of Gerlach, Ely, and Melnick is similar to that of Kemp. It is more sequential (a drawback), but gives separate attention to allocation of time and space, and to the organization of student groups (of proven value for teachers).

It often happens that a teacher has the opportunity to teach a course to several groups of students simultaneously or in successio[n] Time is then available for modification of th[e] system. The author constructed a model re[-] flecting this developmental process. Th[e] model provides for gradual change withou[t] damaging the integrity of the syste[m] (Egnatoff, 1980). It also provides for simula[-] tion which too often takes place in a haphaz[-] ard fashion. The flow chart model is given i[n] Figure 2.

> "Often, in spite of systematic e[f-] fort, the solution falls into plac[e] suddenly and unexpectedly."

Using Kemp's approach, one designs [a] system which will provide an adequate lev[el] of instruction with a modest effor[t.] Although there will be room for improv[e-] ment in such a system, at least the teach[er] will be ready to conduct a satisfactory cour[se] without undue wear and tear on teacher [or] students. The first offering of the course, [(] the first class receiving the course in sim[ul-] taneous presentation) can serve as prototy[pe] for debugging the minimal system. T[he] system can be maintained as is until t[he] teacher is prepared to introduce chang[es.] These can be developed as time permits a[c-] cording to the model of Figure 3. They m[ay] then be introduced into the system as show[n] in Figure 2. Final debugging takes place [in] the contest of the whole system. In this wa[y] the whole system can eventually redefi[ne] itself completely.

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It should be noted that each new compo[n-] ent is designed in relation to the existin[g] system and that at each phase of the develo[p-] ment one keeps checking that too much ti[me] is not being taken from the mainstream [of] teaching and that the component under d[e-] velopment is still of sufficiently high priori[ty.] It should be kept in mind that the mode[ls] presented here are merely formal descri[p-] tions of what many experienced teache[rs] already do as a matter of course.

Course Design

A minimal system was designed to oper[ate] the course and to teach formal problem so[lv-] ing skills. Additional components were th[en] designed and integrated into the system [as] time permitted. Initially a much more elab[or-] ate system was envisaged. The minim[al] system approach served to keep wishf[ul] thinking under control. The minimal syst[em] was implemented as planned. The extent [to] which each additional component was [im-] plemented is indicated in the following li[st.]

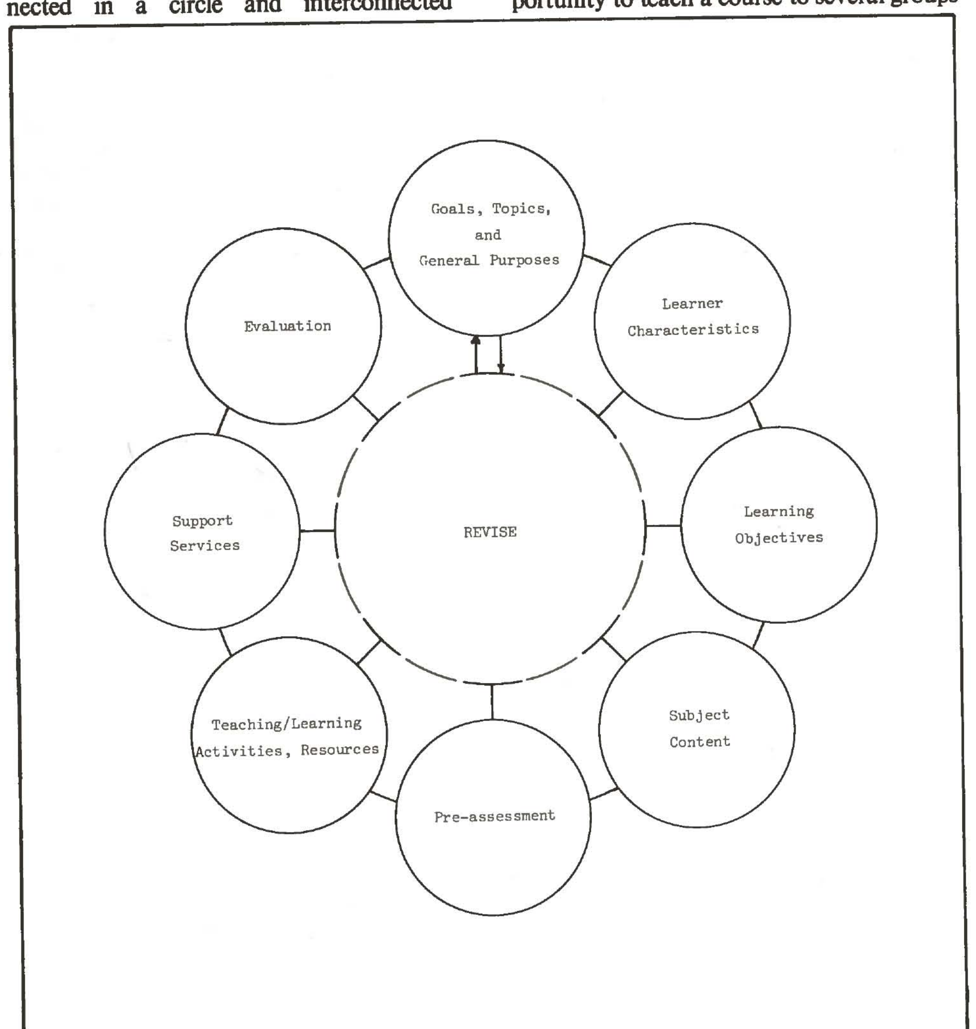

Figure 1. Kemp's model for instructional design. The diagram shows the mode in which goals, topics, and general purposes are being determined. They may need to be changed as a result of learner characteristics, see arrow outward, or may cause changes in subject content, see inward arrow.

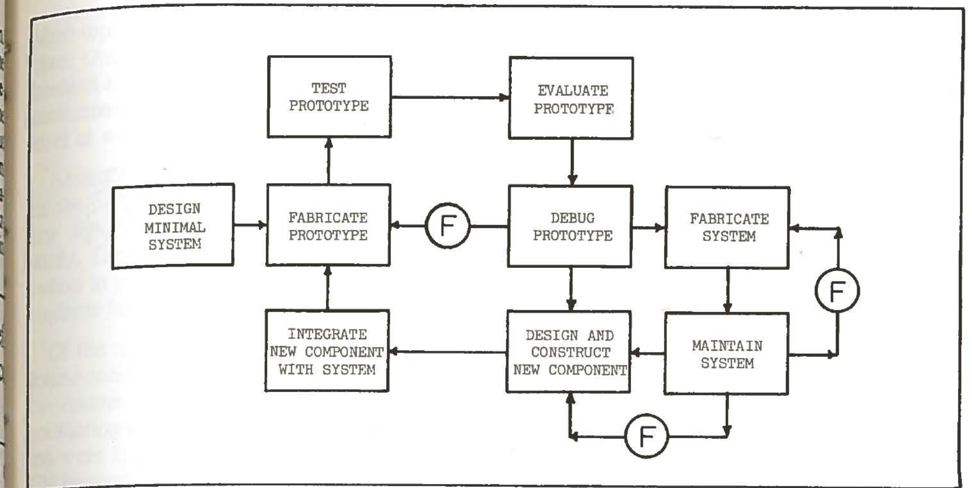

Figure 2. Model for system design and operation.

A. Operating the course Math 205
Minimal system

1. Prepare topical outline and timetable, including test dates.
2. Plan lectures and assignments one or two days in advance. (Pacing of the lectures and assignments was adjusted daily according to difficulties encountered.)
3. Construct tests several days in advance.

Additional components

1. Obtain personal information, relevant to instruction, from each student. (This was done on the first day in written form, and verbally as needed afterwards.)
2. Set up regular optional tutorial periods. (This was done from the beginning, twice a week, in one-and-a-half hour sessions. In response to student requests, additional periods were set up in the morning for those who could not come in the afternoon.)
3. Provide for individual conferences. (The instructor agreed to be available after class and in the afternoon by appointment. A few students took advantage of this and one or two more made use of the telephone. Often brief conferences were held just before or after class, or during break.)
4. Specify instructional objectives and design formative evaluation of learning and teaching accordingly. (This was done informally, but not in written form.)
5. Teach problem solving. (See part B for details.)
6. Provide for individual contracted study projects during the second half of the course. (No student opted for breaking away from the class entirely; however, all students conducted a minor project of their own choice, worth 15 percent of the total mark.)
7. Conduct a written evaluation of the course. (This was done after the midterm exam and resulted in modifications of lectures and assignments.)
8. For each test, construct a sample test and a retest. (The sample tests were given the day before the test and the retest, one or two days afterwards. This was done for the midterm test and the third quiz. All tests were scored by the students immediately after writing them. Sample tests were not always close enough in form and content to the text to provide good guidance. Retests gave students extra opportunity to master content.)

B. Teaching formal problem solving skills
Minimal system

1. Employ Polya's list of questions in lectures, tutorials, and individual conferences.
2. Design a pretest/posttest pair (independent of course content) to measure the learning of problem solving skills. (The pretest was administered on the first day and the posttest one week from the end. There were two comparable sets, A and B, of three problems each. Half of the class used set A as the pretest and set B as the posttest. The sets were switched for the other half of the class. Any significant improvements would thus have to show in both halves of the class.)

Additional Components

1. Keep a journal on lectures and tutorial sessions, including informal feedback.
2. Modify regular (content-oriented) tests to measure the attainment of problem solving objectives. (This was done to a limited extent but the evaluation was obscured by difficulties in understanding the questions.)
3. Set up individual sessions at the beginning and end of the course with a random sample of five or six students, to observe them in the process of problem solving. Make observations based on self-reporting with some prompting from the experimenter. (This type of clinical observation was begun but abandoned. It could have been a rich source of information.)
4. Simulate each lecture with two or three students, one day in advance. For the actual lecture, these students would serve as catalysts and supervisors. They could also assist with individual instruction. (This was done several times, and served to uncover difficulties which were subsequently avoided in the actual lecture. The classroom dynamics in the lecture were very different from those in the simulation. The simulation thus served primarily to provide extra help for the individuals who participated.)
5. Promote self-knowledge of problem solving skills by:
   i) having students write down everything they know about problem solving and expand on this as the course progresses. (This was done early in the course. Each student handed in a summary on a 5 inch by 8 inch file card. The cards were analyzed vis-a-vis Polya's model.)
   ii) commenting on the significance of student responses and questions in class. (This became an integral part of the instructor's interaction with the students.)
6. Study the literature on problem solving and related topics, and incorporate appropriate ideas into the system. (Books and articles added to the author's library over the past decade were supplemented with recommended articles and references. This search method avoided the distractions of a comprehensive search using reference journals and automated data bases.)

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Course Descriptions

The goals of the course, as specified in the course outline given to the students were:

1. To explore algebraic structures;
2. To use the language of mathematics;
3. To distinguish between proof and conjecture;
4. To develop problem solving skills.

The first goal specifies the subject area. The other three all have to do with solving formal problems. The "language" of mathematics provides powerful tools for solving problems which would otherwise be intractable. The third concerns a very important logical distinction essential for evaluating the validity of a solution.

The course spanned a six-week period. It met for 29, two-hour sessions consisting of lectures, discussions, and group and individual work on problems. Tests were also held during these sessions. Optional tutorial sessions were held twice weekly for one and a half hours in the afternoon. Extra sessions were set up on the morning to meet the needs of students who couldn't come in the afternoon. The instructor was generally available for consultation throughout the day. There was no lack of opportunity for the students to get help if they so wished.

"Questions were constructed to require thinking at various cognitive levels."

A set of problems was constructed to accompany each lecture. Some of the problems were worked together or individually in the class session. By student request, the instructor gave an introduction to each problem to increase the students' understanding. It often happened that the sessions deviated from the plan. Assignments were adjusted accordingly. The pace was adjusted carefully at the beginning so that the students were not swamped with new and abstract ideas. The prescribed textbook was abandoned early in the course because of the difficulties students had in reading it. Often, a large portion of the class session was spent dealing with the assignment from the previous day. In this way students were provided with a model of what should go into their solutions. Occasionally problems which proved too difficult were simply dropped. Problems were of three types, those asking to find a result not given, those asking to prove a given result, and those asking to explore (i.e. to create and solve problems on a given topic). Students found the latter two types the most difficult.

At the end of major sections a sheet of problems was given out. The first of these was handled quite well. The second and third sheets were largely exploratory in nature. The students were simply not ready for this type of problem after such a short time span. At the request of the students, solutions were placed in the library, but some students had difficulty even following them. The instructor then selected those problems which he felt were most useful to discuss in detail in class. In some cases, the problems were broken down into simpler problems which the students were able to solve as the discussion proceeded.

Formal testing was done weekly. Each test had a strict time limit. Immediate feedback

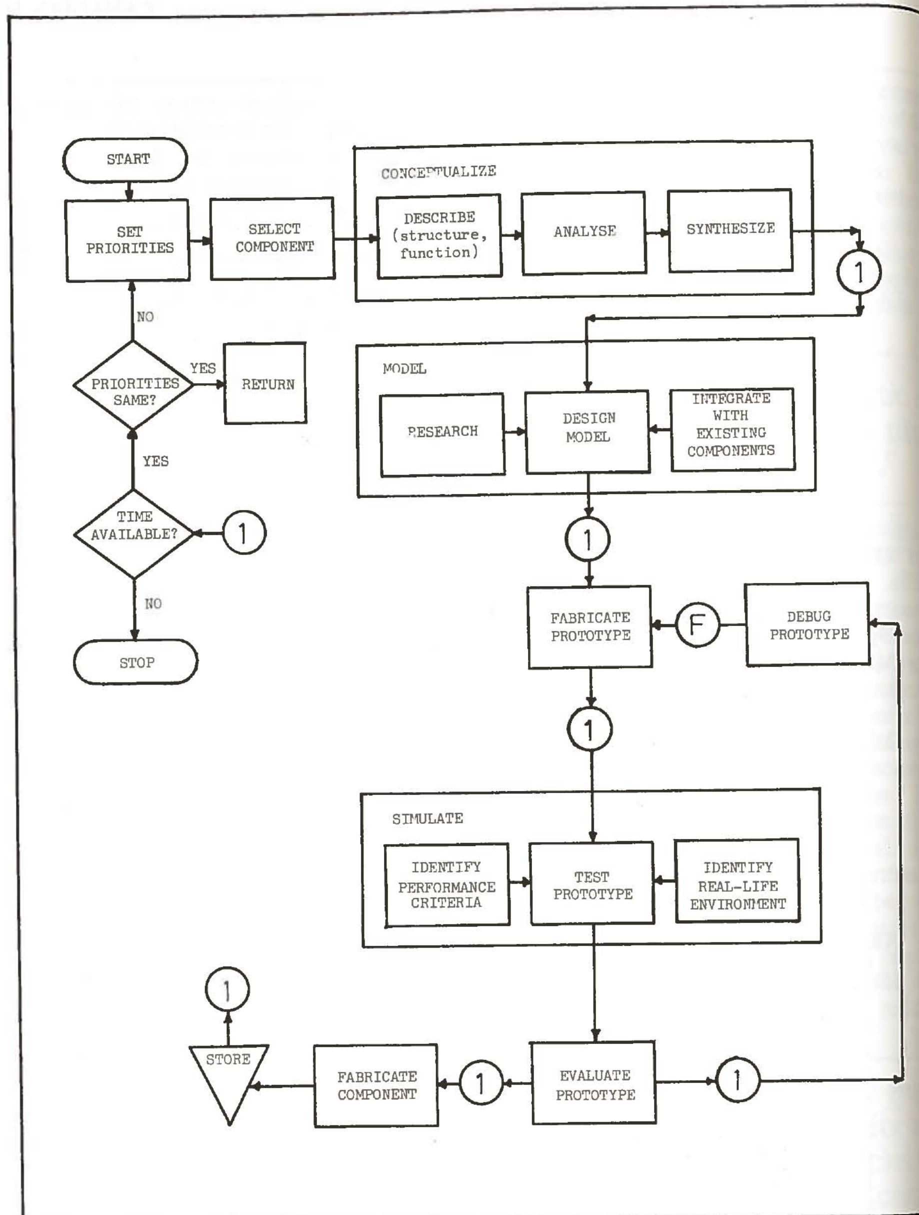

Figure 3. Model for component design and fabrication. The circle containing a 1 indicates branching to check the clock and then appropriate action.

was provided by discussing the solutions after the test was written. Each student scored his own paper. The instructor then conducted his own evaluation and placed the papers in files which were brought to class each day. The files were used in this study to evaluate improvements in problem solving skills. Questions on the tests were similar to

"The greatest work needs to be done in the design of problems."

those of the daily problem assignments. For the midterm test and third quiz, a sample test was given to students one day in advance. Solutions were also provided. The students were advised to use the sample test as a simulation, and to analyse their answers by comparison with those provided. Following the tests, a retest was given for students wh[o] did not do well at first.

The first test was designed by setting ou[t] the objectives and criteria for evaluation. Questions were constructed to require think-ing at various cognitive levels. The same ap-proach was taken informally for subsequent tests. The difficulties students had were no[t] analyzed taxonomically in great depth.

Course topics were chosen from abstract algebra. Basic notions of sets, relations, func-tions, binary operations, groups, rings, and polynomials were studied. Examples were chosen based on familiar concepts. Extensive study was made of modular arithmetic and of symmetry.

At the end of the course students were given free choice of topic for individual study. Assistance was given in finding refer-ences and in structuring the study. Some students consulted the instructor frequently and others worked quite independently.

Most topics were extensions of work done in class. One student studied the arithmetic of boolean algebra and another did an essay on mathematical literacy appropriate for the level at which she would be teaching.

Assigned readings were intended to broaden the perspective of the students, but were not tightly integrated with topics under study. There was insufficient extrinsic motivation to get students started on them. Some students found them interesting and helpful.

Of the sixteen students, all completed the course satisfactorily except one who dropped the course in the first few days because of conflicting commitments. The marks submitted were high (seven A's, six B's, and two C's) because of the mastery approach to evaluation and the keen interest of the students.

Two students were majoring in statistics, and one in pharmacy. The rest were teachers or were working towards certification. All had positive attitudes towards the course. They were appreciative of the adjustments that were made as the course progressed and were frank and open in providing feedback.

Analysis

The instructional design model of Kemp freed the author from working in a single sequence. The author's model for designing and operating the course also proved effective. The model for component design was too detailed to be applied in depth in such a short period. In order to use it effectively one might first determine to what extent existing components could be understood and improved using the model.

The greatest work needs to be done in the design of problems. Students experienced great difficulties understanding definitions and using symbols. They also lacked experience in exploratory work. This made it difficult to start on many of the problems.

The written feedback obtained at midterm was particularly useful. It provided support for the method of instruction and direct information on required changes. Students felt they were mathematically more literate than at the beginning of the course. The assignments were frustrating but the tutorials were helpful.

The schema for problem solving which the students handed in were analyzed in terms of Polya's model. Those students who had the least detail were the weakest students. Most showed concern with understanding the problem but few gave details on planning or evaluating.

There was noticeable improvement in the way some students organized their written work, but it was more difficult to determine to what extent problem-solving ability improved.

In order to test for improvement in ability to solve formal problems, two sets of three problems each were given at the beginning and near the end of the course. Half of the class was given set A as pretest and set B as posttest. The opposite was done for the other half. There were seven students in each group. According to a cursory evaluation of their written answers, nine students did slightly better the first time, one did about as well, and four did better the second time. Of these four, the improvement was very slight for three. The fourth student made little progress on any of the first three problems, but almost completed all three problems on the posttest.

The problems were matched in type on the two tests. Very few students correctly solved the first problem. Those who got it on one set, failed on the other. Four students added to the given on both sets, in order to obtain a problem which they felt was solvable. Several students thought they had complete, correct solutions; however, they had either erred or had left large gaps in reasoning. (These may have been gaps in reporting but not in thinking.) Most students obtained an answer for the second problem on set A, but none gave an argument proving correctness. In the corresponding problem on set B, those who solved the problem correctly (six students) saw the answer, and saw that it was correct, at a glance. (No other possibilities were written down.) Other students failed to understand one of the conditions. A conceptual block arose from applying the condition to individual parts of the problem separately rather than applying it to the whole. About half the students solved the third problem. One student could not get started on this problem in either set. Apparently he was unable to construct a schema for applying the conditions one at a time to eliminate possibilities. Other students had difficulty in interpreting the conditions even though they made a start in the right direction. Of those students who solved the problem completely some kept a record of eliminated possibilities on a three by three matrix. Others appeared to have no need for such an aid. There was a strong correlation between those who solved the third problem in set A and those who solved it in set B.

The pretest/posttest instrument did not prove useful for the intended purpose of determining whether changes had taken place in ability to solve formal problems. Certainly there was no dramatic change. On the other hand it proved useful in testing the problem-solving models used in this study. The author found little difficulty in analyzing the solutions using Polya's questions and Adam's list of conceptual blocks. An excellent framework for diagnosis of the difficulties has thus been acquired. Much work remains to be done on prognosis.

The author would like to incorporate the following elements into the design for teaching problem solving as part of his Grade 10 science course:

1. Having students complete gaps in given solutions (practice in looking for sensible relationships, similar to Cloze procedures in reading);
2. Having students look for errors in given solutions (practice in looking back);
3. Having students analyze and diagnose their own solutions (development of conceptual schemata for problem solving);
4. Presenting students with pairs of similar problems, some related structurally and others only superficially (used for diagnosis of structural thinking);
5. Having students construct their own problems based on given data, conclusion, or other problems (development of exploratory skills through problem finding).

Conclusion

Through this study the author has developed a manageable approach to instructional design. It was particularly satisfying to be able to conduct practical research and to teach simultaneously. Progress was made on a topic of long standing interest. The author is in a much stronger position to teach problem solving. A realistic idea was gained of what can be accomplished in a short time span.

The course was enjoyable and challenging to teach. The students gained exposure to abstract mathematics and an increased awareness of problem solving strategies. There was good rapport amongst the students and between students and instructor. This made it easy to adjust plans as the course evolved.

The major problem which came into focus was the need to move away from a blind trial-and-error approach to problem solving and towards an approach based on sensible, productive processes.

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